ENGINEERING MATHEMATICS

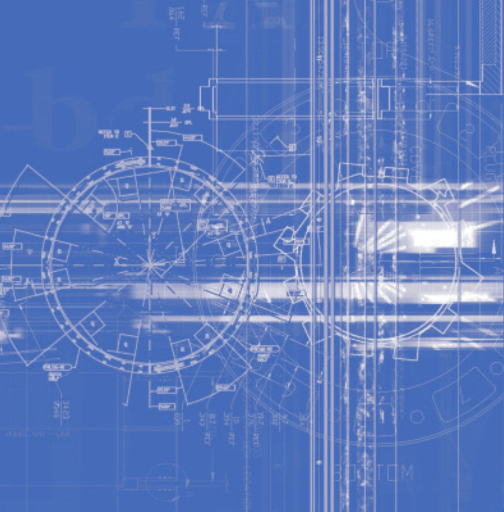
A Foundation for Electronic, Electrical, Communications and Systems Engineers

FIFTH EDITION

Anthony Croft • Robert Davison
Martin Hargreaves • James Flint

Pearson

Engineering Mathematics



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To Kate, Tom and Harvey – A.C.

To Kathy – R.D.

To my father and mother – M.H.

To Suzanne, Alexandra and Dominic – J.F.

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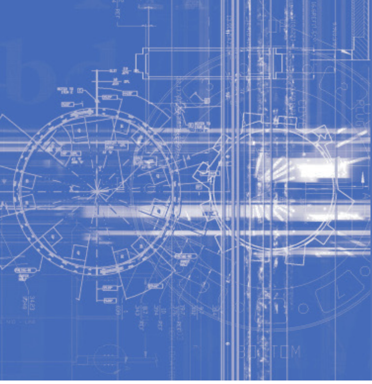

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Preface

Audience

This book has been written to serve the mathematical needs of students engaged in a first course in engineering at degree level. It is primarily aimed at students of electronic, electrical, communications and systems engineering. Systems engineering typically encompasses areas such as manufacturing, control and production engineering. The textbook will also be useful for engineers who wish to engage in self-study and continuing education.

Motivation

Engineers are called upon to analyse a variety of engineering systems, which can be anything from a few electronic components connected together through to a complete factory. The analysis of these systems benefits from the intelligent application of mathematics. Indeed, many cannot be analysed without the use of mathematics. Mathematics is the language of engineering. It is essential to understand how mathematics works in order to master the complex relationships present in modern engineering systems and products.

Aims

There are two main aims of the book. Firstly, we wish to provide an accessible, readable introduction to engineering mathematics at degree level. The second aim is to encourage the integration of engineering and mathematics.

Content

The first three chapters include a review of some important functions and techniques that the reader may have met in previous courses. This material ensures that the book is self-contained and provides a convenient reference.

Traditional topics in algebra, trigonometry and calculus have been covered. Also included are chapters on set theory, sequences and series, Boolean algebra, logic, difference equations and the z transform. The importance of signal processing techniques is reflected by a thorough treatment of integral transform methods. Thus the Laplace, z and Fourier transforms have been given extensive coverage.

In the light of feedback from readers, new topics and new examples have been added in the fifth edition. Recognizing that motivation comes from seeing the applicability of mathematics we have focused mainly on the enhancement of the range of applied examples. These include topics on the discrete cosine transform, image processing, applications in music technology, communications engineering and frequency modulation.

Style

The style of the book is to develop and illustrate mathematical concepts through examples. We have tried throughout to adopt an informal approach and to describe mathematical processes using everyday language. Mathematical ideas are often developed by examples rather than by using abstract proof, which has been kept to a minimum. This reflects the authors' experience that engineering students learn better from practical examples, rather than from formal abstract development. We have included many engineering examples and have tried to make them as free-standing as possible to keep the necessary engineering prerequisites to a minimum. The engineering examples, which have been carefully selected to be relevant, informative and modern, range from short illustrative examples through to complete sections which can be regarded as case studies. A further benefit is the development of the link between mathematics and the physical world. An appreciation of this link is essential if engineers are to take full advantage of engineering mathematics. The engineering examples make the book more colourful and, more importantly, they help develop the ability to see an engineering problem and translate it into a mathematical form so that a solution can be obtained. This is one of the most difficult skills that an engineer needs to acquire. The ability to manipulate mathematical equations is by itself insufficient. It is sometimes necessary to derive the equations corresponding to an engineering problem. Interpretation of mathematical solutions in terms of the physical variables is also essential. Engineers cannot afford to get lost in mathematical symbolism.

Format

Important results are highlighted for easy reference. Exercises and solutions are provided at the end of most sections; it is essential to attempt these as the only way to develop competence and understanding is through practice. A further set of review exercises is provided at the end of each chapter. In addition some sections include exercises that are intended to be carried out on a computer using a technical computing language such as MATLAB®, GNU Octave, Mathematica or Python®. The MATLAB® command syntax is supported in several software packages as well as MATLAB® itself and will be used throughout the book.

Supplements

A comprehensive Solutions Manual is obtainable free of charge to lecturers using this textbook. It is also available for download via the web at *www.pearsoned.co.uk/croft*.

Finally we hope you will come to share our enthusiasm for engineering mathematics and enjoy the book.

Anthony Croft
Robert Davison
Martin Hargreaves
James Flint
March 2017

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Tables

Table 29.7 from *Biometrika Tables for Statisticians*, Vol. 1, New York: Holt, Rinehart & Winston (Hays, W.L. and Winkler, R.L. 1970) Table 1, © Cambridge University Press.

Text

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1 Review of algebraic techniques

Contents

1.1 INTRODUCTION

This chapter introduces some algebraic techniques which are commonly used in engineering mathematics. For some readers this may be revision. Section 1.2 examines the laws of indices. These laws are used throughout engineering mathematics. Section 1.3 looks at number bases. Section 1.4 looks at methods of solving polynomial equations. Section 1.5 examines algebraic fractions, while Section 1.6 examines the solution of inequalities. Section 1.7 looks at partial fractions. The chapter closes with a study of summation notation.

Computers are used extensively in all engineering disciplines to perform calculations. Some of the examples provided in this book make use of the technical computing language **MATLAB®**, which is commonly used in both an academic and industrial setting.

Because MATLAB® and many other similar languages are designed to compute not just with single numbers but with entire sequences of numbers at the same time, data is entered in the form of **arrays**. These are multi-dimensional objects. Two particular types of array are **vectors** and **matrices** which are studied in detail in Chapters 7 and 8.

Apart from being able to perform basic mathematical operations with vectors and matrices, MATLAB® has, in addition, a vast range of built-in computational functions which are straightforward to use but nevertheless are very powerful. Many of these high-level functions are accessible by passing data to them in the form of vectors and matrices. A small number of these special functions are used and explained in this text. However, to get the most out of a technical computing language it is necessary to develop a good understanding of what the software can do and to make regular reference to the manual.

1.2 LAWS OF INDICES

Consider the product $6 \times 6 \times 6 \times 6 \times 6$. This may be written more compactly as 6^5. We call 5 the **index** or **power**. The **base** is 6. Similarly, $y \times y \times y \times y$ may be written as y^4. Here the base is y and the index is 4.

Example 1.1 Write the following using index notation:

(a) $(-2)(-2)(-2)$ (b) 4.4.4.5.5 (c) $\dfrac{yyy}{xxxx}$ (d) $\dfrac{aa(-a)(-a)}{bb(-b)}$

Solution (a) $(-2)(-2)(-2)$ may be written as $(-2)^3$.

(b) 4.4.4.5.5 may be written as $4^3 5^2$.

(c) $\dfrac{yyy}{xxxx}$ may be written as $\dfrac{y^3}{x^4}$.

(d) Note that $(-a)(-a) = aa$ since the product of two negative quantities is positive. So $aa(-a)(-a) = aaaa = a^4$. Also $bb(-b) = -bbb = -b^3$. Hence

$$\frac{aa(-a)(-a)}{bb(-b)} = \frac{a^4}{-b^3} = -\frac{a^4}{b^3}$$

Example 1.2 Evaluate

(a) 7^3 (b) $(-3)^3$ (c) $2^3(-3)^4$

Solution (a) $7^3 = 7.7.7 = 343$

(b) $(-3)^3 = (-3)(-3)(-3) = -27$

(c) $2^3(-3)^4 = 8(81) = 648$

Most scientific calculators have an x^y button to enable easy calculation of expressions of a similar form to those in Example 1.2.

1.2.1 Multiplying expressions involving indices

Consider the product $(6^2)(6^3)$. We may write this as

$$(6^2)(6^3) = (6.6)(6.6.6) = 6^5$$

So

$$6^2 6^3 = 6^5$$

This illustrates the **first law of indices** which is

$$a^m a^n = a^{m+n}$$

When expressions with the same base are multiplied, the indices are added.

Example 1.3 Simplify each of the following expressions:

(a) $3^9 3^{10}$ (b) $4^3 4^4 4^6$ (c) $x^3 x^6$ (d) $y^4 y^2 y^3$

Solution

(a) $3^9 3^{10} = 3^{9+10} = 3^{19}$

(b) $4^3 4^4 4^6 = 4^{3+4+6} = 4^{13}$

(c) $x^3 x^6 = x^{3+6} = x^9$

(d) $y^4 y^2 y^3 = y^{4+2+3} = y^9$

Engineering application 1.1

Power dissipation in a resistor

The **resistor** is one of the three fundamental electronic components. The other two are the capacitor and the inductor, which we will meet later. The role of the resistor is to reduce the current flow within the branch of a circuit for a given voltage. As current flows through the resistor, electrical energy is converted into heat. Because the energy is lost from the circuit and is effectively wasted, it is termed **dissipated** energy. The rate of energy dissipation is known as the power, P, and is given by

$$P = I^2 R \qquad (1.1)$$

where I is the current flowing through the resistor and R is the resistance value. Note that the current is raised to the power 2. Note that power, P, is measured in watts; current, I, is measured in amps; and resistance, R, is measured in ohms.

There is an alternative formula for power dissipation in a resistor that uses the voltage, V, across the resistor. To obtain this alternative formula we need to use **Ohm's law**, which states that the voltage across a resistor, V, and the current passing through it, are related by the formula

$$V = IR \qquad (1.2)$$

From Equation (1.2) we see that

$$I = \frac{V}{R} \qquad (1.3)$$

Combining Equations (1.1) and (1.3) gives

$$P = \left(\frac{V}{R}\right)^2 R = \frac{V}{R} \cdot \frac{V}{R} \cdot R = \frac{V^2}{R}$$

→

Note that in this formula for P, the voltage is raised to the power 2. Note an important consequence of this formula is that doubling the voltage, while keeping the resistance fixed, results in the power dissipation increasing by a factor of 4, that is 2^2. Also trebling the voltage, for a fixed value of resistance, results in the power dissipation increasing by a factor of 9, that is 3^2.

Similar considerations can be applied to Equation 1.1. For a fixed value of resistance, doubling the current results in the power dissipation increasing by a factor of 4, and trebling the current results in the power dissipation increasing by a factor of 9.

Consider the product $3(3^3)$. Now

$$3(3^3) = 3(3.3.3) = 3^4$$

Also, using the first law of indices we see that $3^1 3^3 = 3^4$. This suggests that 3 is the same as 3^1. This illustrates the general rule:

$$a = a^1$$

Raising a number to the power 1 leaves the number unchanged.

Example 1.4 Simplify (a) $5^6 5$ (b) $x^3 x x^2$

Solution (a) $5^6 5 = 5^{6+1} = 5^7$ (b) $x^3 x x^2 = x^{3+1+2} = x^6$

1.2.2 Dividing expressions involving indices

Consider the expression $\dfrac{4^5}{4^3}$:

$$\begin{aligned}\frac{4^5}{4^3} &= \frac{4.4.4.4.4}{4.4.4} \\ &= 4.4 \qquad \text{by cancelling 4s} \\ &= 4^2\end{aligned}$$

This serves to illustrate the **second law of indices** which is

$$\frac{a^m}{a^n} = a^{m-n}$$

When expressions with the same base are divided, the indices are subtracted.

Example 1.5 Simplify

(a) $\dfrac{5^9}{5^7}$ (b) $\dfrac{(-2)^{16}}{(-2)^{13}}$ (c) $\dfrac{x^9}{x^5}$ (d) $\dfrac{y^6}{y}$

Solution (a) $\dfrac{5^9}{5^7} = 5^{9-7} = 5^2$

(b) $\dfrac{(-2)^{16}}{(-2)^{13}} = (-2)^{16-13} = (-2)^3$

(c) $\dfrac{x^9}{x^5} = x^{9-5} = x^4$

(d) $\dfrac{y^6}{y} = y^{6-1} = y^5$

Consider the expression $\dfrac{2^3}{2^3}$. Using the second law of indices we may write

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

But, clearly, $\dfrac{2^3}{2^3} = 1$, and so $2^0 = 1$. This illustrates the general rule:

$$a^0 = 1$$

Any expression raised to the power 0 is 1.

1.2.3 Negative indices

Consider the expression $\dfrac{4^3}{4^5}$. We can write this as

$$\frac{4^3}{4^5} = \frac{4.4.4}{4.4.4.4.4} = \frac{1}{4.4} = \frac{1}{4^2}$$

Alternatively, using the second law of indices we have

$$\frac{4^3}{4^5} = 4^{3-5} = 4^{-2}$$

So we see that

$$4^{-2} = \frac{1}{4^2}$$

Thus we are able to interpret negative indices. The sign of an index changes when the expression is inverted. In general we can state

$$a^{-m} = \frac{1}{a^m} \qquad a^m = \frac{1}{a^{-m}}$$

Example 1.6 Evaluate the following:

(a) 3^{-2} (b) $\dfrac{2}{4^{-3}}$ (c) 3^{-1} (d) $(-3)^{-2}$ (e) $\dfrac{6^{-3}}{6^{-2}}$

Solution

(a) $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

(b) $\dfrac{2}{4^{-3}} = 2(4^3) = 2(64) = 128$

(c) $3^{-1} = \dfrac{1}{3^1} = \dfrac{1}{3}$

(d) $(-3)^{-2} = \dfrac{1}{(-3)^2} = \dfrac{1}{9}$

(e) $\dfrac{6^{-3}}{6^{-2}} = 6^{-3-(-2)} = 6^{-1} = \dfrac{1}{6^1} = \dfrac{1}{6}$

Example 1.7 Write the following expressions using only positive indices:

(a) x^{-4} (b) $3x^{-4}$ (c) $\dfrac{x^{-2}}{y^{-2}}$ (d) $3x^{-2}y^{-3}$

Solution

(a) $x^{-4} = \dfrac{1}{x^4}$

(b) $3x^{-4} = \dfrac{3}{x^4}$

(c) $\dfrac{x^{-2}}{y^{-2}} = x^{-2}y^2 = \dfrac{y^2}{x^2}$

(d) $3x^{-2}y^{-3} = \dfrac{3}{x^2y^3}$

Engineering application 1.2

Power density of a signal transmitted by a radio antenna

A **radio antenna** is a device that is used to convert electrical energy into electromagnetic radiation, which is then transmitted to distant points.

An ideal theoretical point source radio antenna which radiates the same power in all directions is termed an **isotropic** antenna. When it transmits a radio wave, the wave spreads out equally in all directions, providing there are no obstacles to block the expansion of the wave. The power generated by the antenna is uniformly distributed on the surface of an expanding sphere of area, A, given by

$$A = 4\pi r^2$$

where r is the distance from the generating antenna to the wave front.

The **power density**, S, provides an indication of how much of the signal can potentially be received by another antenna placed at a distance r. The actual power received depends on the effective area or aperture of the antenna, which is usually expressed in units of m^2.

Electromagnetic field exposure limits for humans are sometimes specified in terms of a power density. The closer a person is to the transmitter, the higher the power density will be. So a safe distance needs to be determined.

The power density is the ratio of the power transmitted, P_t, to the area over which it is spread

$$S = \frac{\text{power transmitted}}{\text{area}} = \frac{P_\text{t}}{4\pi r^2} = \frac{P_\text{t}}{4\pi} r^{-2} \quad \text{W m}^{-2}$$

Note that r in this equation has a **negative index**. This type of relationship is known as an **inverse square law** and is found commonly in science and engineering.

Note that if the distance, r, is doubled, then the area, A, increases by a factor of 4 (i.e. 2^2). If the distance is trebled, the area increases by a factor of 9 (i.e. 3^2) and so on. This means that as the distance from the antenna doubles, the power density, S, decreases to a quarter of its previous value; if the distance trebles then the power density is only a ninth of its previous value.

1.2.4 Multiple indices

Consider the expression $(4^3)^2$. This may be written as

$$(4^3)^2 = 4^3 . 4^3 = 4^{3+3} = 4^6$$

This illustrates the **third law of indices** which is

$$(a^m)^n = a^{mn}$$

Note that the indices m and n have been multiplied.

Example 1.8 Write the following expressions using a single index:

(a) $(3^2)^4$ (b) $(7^{-2})^3$ (c) $(x^2)^{-3}$ (d) $(x^{-2})^{-3}$

Solution

(a) $(3^2)^4 = 3^{2\times 4} = 3^8$

(b) $(7^{-2})^3 = 7^{-2\times 3} = 7^{-6}$

(c) $(x^2)^{-3} = x^{2\times(-3)} = x^{-6}$

(d) $(x^{-2})^{-3} = x^{-2\times -3} = x^6$

Consider the expression $(2^4 5^2)^3$. We see that

$$\begin{aligned}(2^4 5^2)^3 &= (2^4 5^2)(2^4 5^2)(2^4 5^2)\\ &= 2^4 2^4 2^4 5^2 5^2 5^2\\ &= 2^{12} 5^6\end{aligned}$$

This illustrates a generalization of the third law of indices which is

$$(a^m b^n)^k = a^{mk} b^{nk}$$

Example 1.9 Remove the brackets from

(a) $(2x^2)^3$ (b) $(-3y^4)^2$ (c) $(x^{-2}y)^3$

Solution

(a) $(2x^2)^3 = (2^1 x^2)^3 = 2^3 x^6 = 8x^6$

(b) $(-3y^4)^2 = (-3)^2 y^8 = 9y^8$

(c) $(x^{-2}y)^3 = x^{-6}y^3$

Engineering application 1.3

Radar scattering

It has already been shown in Engineering application 1.2 that the power density of an isotropic transmitter of radio waves is

$$S = \frac{P_t}{4\pi} r^{-2} \text{ W m}^{-2}$$

It is possible to use radio waves to detect distant objects. The technique involves transmitting a radio signal, which is then reflected back when it strikes a target. This weak reflected signal is then picked up by a receiving antenna, thus allowing a number of properties of the target to be deduced, such as its angular position and distance from the transmitter. This system is known as **radar**, which was originally an acronym standing for **RA**dio **D**etection **A**nd **R**anging.

When the wave hits the target it produces a quantity of reflected power. The power depends upon the object's **radar cross-section** (RCS), normally denoted by the Greek lower case letter sigma, σ, and having units of m^2. The power reflected at the object, P_r, is given by

$$P_r = S\sigma = \frac{P_t\sigma}{4\pi}r^{-2} \text{ W}$$

Some military aircraft use special techniques to minimize the RCS in order to reduce the amount of power they reflect and hence minimize the chance of being detected.

If the reflected power at the target is assumed to spread spherically, when it returns to the transmitter position it will have the power density, S_r, given by

$$S_r = \frac{\text{power reflected at target}}{\text{area}} = \frac{P_r}{4\pi}r^{-2} \text{ W m}^{-2}$$

Substituting for the reflected power, P_r, gives

$$S_r = \frac{\text{power reflected at target}}{\text{area}} = \frac{\left(\frac{P_t\sigma}{4\pi}r^{-2}\right)}{4\pi}r^{-2} = \frac{P_t\sigma}{4\pi \times 4\pi}\left(r^{-2}\right)^2$$

$$= \frac{P_t\sigma}{(4\pi)^2}r^{-4} \text{ W m}^{-2}$$

Note that the product of the two r^{-2} terms has been calculated using the third law of indices.

This example illustrates one of the main challenges with radar design which is that the power density returned by a distant object is very much smaller than the transmitted power, even for targets with a large RCS. For theoretical isotropic antennas, the received power density depends upon the factor r^{-4}. This factor diminishes rapidly for large values of r, that is, as the object being detected gets further away.

In practice, the transmit antennas used are not isotropic but **directive** and often scan the area of interest. They also make use of receive antennas with a large effective area which can produce a viable signal from the small reflected power densities.

1.2.5 Fractional indices

The third law of indices states that $(a^m)^n = a^{mn}$. If we take $a = 2$, $m = \frac{1}{2}$ and $n = 2$ we obtain

$$(2^{1/2})^2 = 2^1 = 2$$

So when $2^{1/2}$ is squared, the result is 2. Thus, $2^{1/2}$ is a square root of 2. Each positive number has two square roots and so

$$2^{1/2} = \sqrt{2} = \pm 1.4142\ldots$$

Similarly

$$(2^{1/3})^3 = 2^1 = 2$$

so that $2^{1/3}$ is a cube root of 2:

$$2^{1/3} = \sqrt[3]{2} = 1.2599\ldots$$

In general $2^{1/n}$ is an nth root of 2. The general law states

$x^{1/n}$ is an nth root of x

Example 1.10 Write the following using a single positive index:

(a) $(3^{-2})^{1/4}$ (b) $x^{2/3}x^{5/3}$ (c) $yy^{-2/5}$ (d) $\sqrt{k^3}$

Solution

(a) $(3^{-2})^{1/4} = 3^{-2\times\frac{1}{4}} = 3^{-1/2} = \dfrac{1}{3^{1/2}}$

(b) $x^{2/3}x^{5/3} = x^{2/3+5/3} = x^{7/3}$

(c) $yy^{-2/5} = y^1y^{-2/5} = y^{1-2/5} = y^{3/5}$

(d) $\sqrt{k^3} = (k^3)^{1/2} = k^{3\times\frac{1}{2}} = k^{3/2}$

Example 1.11 Evaluate

(a) $8^{1/3}$ (b) $8^{2/3}$ (c) $8^{-1/3}$ (d) $8^{-2/3}$ (e) $8^{4/3}$

Solution We note that 8 may be written as 2^3.

(a) $8^{1/3} = (2^3)^{1/3} = 2^1 = 2$

(b) $8^{2/3} = (8^{1/3})^2 = 2^2 = 4$

(c) $8^{-1/3} = \dfrac{1}{8^{1/3}} = \dfrac{1}{2}$

(d) $8^{-2/3} = \dfrac{1}{8^{2/3}} = \dfrac{1}{4}$

(e) $8^{4/3} = (8^{1/3})^4 = 2^4 = 16$

Engineering application 1.4

Skin depth in a radial conductor

When an alternating current signal travels along a conductor, such as a copper wire, most of the current is found near the surface of the conductor. Nearer to the centre of the conductor, the current diminishes. The depth of penetration of the signal, termed the **skin depth**, into the conductor depends on the frequency of the signal. Skin depth, illustrated in Figure 1.1, is defined as the depth at which the current

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